

## Alternatives available to Ofwat in making comparisons

### Introduction

1. Section 6 of our report discusses alternative methodologies available to Ofwat in making comparisons. This appendix presents some relevant details.
2. Ofwat's main econometric technique is currently 'corrected' ordinary least-squares regression (COLS). The main alternatives to the COLS analysis are stochastic frontier analysis (SFA) and data envelopment analysis (DEA).

### Stochastic Frontier Analysis

3. SFA makes assumptions about the distribution of residuals. These assumptions allow SFA to differentiate between the element of the residual which is due to efficiency and the element which is due to measurement error. In the absence of measurement error, all SFA residuals will be positive and in a traditional formulation SFA assumes that most will be small and only a few will be large. An asymmetric function—such as 'half-normal' (ie half a bell curve)—is used to model the distribution of such residuals. However, if observations depart from this distribution, SFA assumes that the additional variation in residuals is due to measurement error in costs and further assumes that the measurement error is normally distributed (ie distributed as a bell curve).
4. SFA employs the method of maximum likelihood to estimate the respective contributions of inefficiency and measurement error in the model's residuals.

### Data Envelopment Analysis

5. DEA does not include an allowance for measurement error. DEA compares each company with its closest peers. It does so by envisaging a composite company made up of proportions of other companies. A DEA-efficient company is one that satisfies two conditions:<sup>1</sup>
  - its costs are no higher than a composite company with the same level of cost drivers; and
  - its cost drivers exceed or equal those of any composite company with the same costs.
6. These conditions define a mathematical optimization problem and its solution provides information on each company's performance and identifies its closest comparators. The most efficient companies cannot be bettered by their peers and they define the DEA-efficient frontier. The DEA frontier will not be a smooth curve (or curved surface) because it is formed by tracing an outline that connects by straight lines (or planes) the DEA-efficient companies.

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<sup>1</sup>These conditions essentially define a linear optimization problem for each company. The DEA-efficient frontier is the piecewise linear envelope that encompasses the DEA-efficient companies.

7. When parameters are estimated statistically, it is possible to place confidence intervals around the parameter estimates and the efficiency frontier. The robustness of the DEA boundary to variations in the input data could be tested using post-'estimation' sensitivity analysis, although this is less straightforward than the computation of confidence intervals. The sensitivity analysis would show whether the DEA-efficient companies remain so when the input data change by small amounts.